

Image Processing: Stochastic Model Based Approach

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Preface

Digital Image analyses at low-level is a complicated task, because it has the responsibility of analysing the images at micro-level and in-depth. The intention of this book is that how to effectively and efficiently perform the model based image processing tasks at low-level. Attention has been focused on the concepts of sampling, quantization, randomness, and how to mathematically characterize and model an image, and how to effectively utilize the model to perform various advanced image processing tasks such as *Texture Analyses, Edge detection, Compression, Restoration* by a single model. This book focuses on construction of a stochastic model, which is coined as Full Range Gaussian Markov Random Field model, and also illustrates the efficiency and effectiveness of the model based work. This book discusses the concept of Bayesian methodology and how to incorporate the prior information in various image processing topics, in neat and simple way. The way of discussion and the concept provided are very useful and draw the attention of the postgraduate students, researchers, scientists and engineers to design image processing systems and perform research at advanced level in the newly emerging topics. This book also illustrates the research concepts with a number of examples. Especially, this book is useful to the researchers, because some novel concepts at research level have been introduced, and also more than 250 citations have been incorporated from various scholarly published articles.

The advent of the imaging science and technology, and its applications help the various fields, namely medicine, defence, remote sensing, robot vision, pattern recognition, traffic, forensic science etc. to flourish with advanced technology and fast growth. Especially in the field of medicine, remote sensing and pattern recognition, image processing plays significant role. The outcome of the image analysis may be either an image or a set of attributes or parameters related to the image. Image processing is a sub-class of signal processing and also

it is a discrete space with time dependent, so it is convenient to model the images using mathematical or statistical models such as Wavelet, time series, stochastic models etc.

This book discusses the various fundamental image processing methods based on stochastic model, which is coined as Full Range Gaussian Markov Random Field model. This book explains the concepts, randomness, sampling and quantization, and how the mathematical model is designed and constructed according to the nature of the images, in neat and simple manner. Also, the book discusses the advantages and disadvantages of the various parameter estimation techniques in the context of the image processing. Though, this book does not cover a more number of topics in image processing, the covered some topics are very useful to the readers at advanced level. The book is organised with eight chapters follows.

Chapter 1, introduces the basic concepts and various applications of the image processing, and also elaborates the different types of image processing models such as *causal*, *semi-causal*, *non-causal*. Chapter 2 discusses the concepts of randomness, sampling and quantization in the context of image processing.

Chapter 3, introduces the stochastic model and its appropriateness in the image analysis at low-level. Also it narrates the Bayesian concepts and parameter estimation technique. Chapter 4 discusses texture analysis such as texture characterization, identification, representation and description. Chapter 5 discusses the identification and distinguishing the textures from structures and edge detection. Chapter 6 and 7 discusses the compression and restoration of the damaged regions in the images, and Chapter 8 concludes with conclusion. Each chapter provides a summary and annotated bibliography.

Chapter 1

Introduction

1.1 Background

Since 1960s, the researchers in computer vision aggressively concentrated on image processing and their applications in various fields such as video conferencing, telemedicine, remote sensing via satellite, forensic science, agricultural, education, defence, news and current affairs etc. To fulfil all these requirements, it is required to analyse the content of an image in depth. With the advent of the advanced technologies in computer vision, advancements in the storage media and electronic communication channels; above all these things, the invention and discovery of the various mathematical and statistical techniques to effectively process or analyse the images becomes more effective for the evolution of the image processing. Digital Image processing enables the reversible, virtually noise-free modification of an image in the form of a lattice of integers instead of the classical darkroom manipulations or filtration of time dependent voltages necessary for analog images and video signals. In order to understand the contents of an image, various tasks are performed on it such as image understanding, image processing and image analysis.

Image understanding is concerned with symbolic descriptions and structure, namely, image formation, surface orientation, image intensity, gradient space and reflectivity function. In the context of digital image processing, texture plays noteworthy role in image formation, even if it is texture image or structure image it is formed by the textures. Simply, we can say that the images cannot be formed without

textures. The Nature of the images is determined by the structure and orientation of the textures. Based on the textures, an image can be analysed, characterized, identified, described, and represented. Image intensities and the texture features are considered for segmentation of an image into various regions or edge-fragments. The gradient space is exploited to extract the edges, curves, boundaries etc. These types of image understanding tools are effectively utilised for analyses and processing of an image. In order to perform properly the image processing and image analysis tasks, it is necessary to understand how the images are formed, what determines the observed intensity in the image and the structure of the images.

Image processing deals deterministic and stochastic representation of images, i.e. is image transforms and image models. It also concerned with image data compression and improving the quality of the image by filtering and by removing any degradations present in the image, viz. image enhancement and image restoration. On the other hand, the image analysis deals with the tasks like extraction of lines, curves and regions in images, classification and segmentation of objects in the image using boundary information, texture analysis, analysis of a sequence of images with the interest of estimating the motion of objects and scene analysis. Hence, in general, in image processing, the inputs and outputs are images while in image analysis, the outputs are a list of objects present in the image or a set of features such as edges, curves and boundaries.

The image processing and image analysis are conducted in two stages, namely, low-level and high-level. The low-level is concerned with image representation, classification, segmentation, object recognition, compression, edge detection and texture analysis, whereas in the high-level stage, we regard it as an interpretation of the results obtained at the low-level stage. The low-level stage involved with models that contain classical knowledge about image formation and object recognition that are independent of the class of images under analysis. The high-level stage is involved with interpretations of applications of specific knowledge about the concept of the

scene analysis. The images are analysed in low-level stage by various methods, which are obtained by using either edge-based or region-based approach. Though the region-based approach and edge-based approaches are complementary to each other the edge-based approach has been used widely. Using the edge-based approach, a number of methods have been proposed for low-level analysis viz. image compression, classification, segmentation and pattern recognition. These methods can be grouped into two classes. In one class, the methods are proposed based on the underlying assumption about the uniformity of intensities in local image regions. The images, which are assumed to be consisting of local regions with uniform intensities are called untextured images. The images of real objects often do not exhibit local regions of uniform intensities due to differences in object surface properties like roughness, orientation, reflectance levels, etc. The former methods of classification and segmentation are not applicable in the latter class of images. The latter class of images is called texture images. Another class of methods, which are usually known as texture analysis methods have been proposed for low-level classification and segmentation of textured images.

In general, the mathematical and statistical model based techniques play a significant role in image processing. Generally, the fundamental image processing methods can be broadly classified into image acquisition, representation and description, pre-processing, enhancement and filtering, edge detection, restoration, classification, segmentation, morphological analysis, compression, and object recognition. To perform these fundamental image processing methods, so many mathematical and statistical techniques are available. This book discusses only the model based image processing and analyses.

Almost all the image processing and analysis have been performed under any one of the following techniques: (i) Transform based and (ii) model based.

The transform based techniques are widely used in image processing such as image coding and restoration.

Various transform techniques and their efficiency have been reported in the literature. Among the existing transform techniques, the most widely used are Discrete Fourier Transform (DFT), Discrete Cosine Transform (DCT), Discrete Sine Transform (DST), Karhunen-Loeve Transform (KLT), Walsh-Hadamard Transform, Harr Transform and Orthogonal Polynomial Transform. Each transform technique has its own specific features, which are discussed in detail in section 5.1 of Chapter 5. In recent years, the last one or two decades, the transform based Wavelets and Fractal image compression techniques becoming popular due to its efficiency. Most of the transform based techniques demand more computations and some of them require large amount of memory. Also, the applications of the transform based techniques are limited at low-level image processing and analysis when compared to that of model based techniques.

The model based techniques are most appropriate to effectively handle the problems involved with large amount of data like image filtering, object recognition, etc. To handle this volume of data, it would be preferable to have an underlying model that explains the dominant statistical characteristics of the given data. The different classes of models have been suggested in the literature, which exploit the statistical properties among the neighbouring pixels for low-level image processing or analysis. The statistical models attracted many researchers, due to its wide range of applications at low-level image processing and analysis such as texture analysis, smoothing, enhancement, restoration, segmentation, edge detection, image data compression, etc.

The statistical models are becoming increasingly important because of their role in the development of useful algorithms for image processing and analysis. It is observed that most of the applications of image processing use some sort of statistical models. Generally, the models are not usually made explicit, but are made implicit by the adoption of assumptions that incorporate certain model assumptions within them. Most of the algorithms, which use the assumption that the image can be treated as a random process with wide sense of stationary properties, linear

dependency, white noise are uncorrelated. In that sense, Markov Random Field (MRF) and Autoregressive (AR) models are most appropriate for almost all the low-level image processing. Many researchers have explored the efficiency of the MRF and AR models for low-level image processing such as image smoothing, object recognition, classification, segmentation, texture representation, texture synthesis, compression and reconstruction, etc. Most of the images satisfy the properties of MRF and AR models. The pixels in a two-dimensional image are spatially equal interval of distance in row wise and column wise as in time series (equal interval of time) and the pixels in the images satisfy the sampling properties and it satisfies stationarity, linear dependency, white noise uncorrelated. Hence the MRF and AR models have drawn the attention of the many researchers in different fields of image processing and analysis. As discussed in Chapter 1, different types of stochastic models used for image processing and analysis are reported by many authors, that include, Autoregressive (AR), Moving Average (MA), Autoregressive Moving Average (ARMA) and Autoregressive Integrated Moving Average (ARIMA) with various assumptions about the image. The assumptions are Random Field, Markov Random Field, Gibbs Field and σ -Field etc. are made on the basis of nature of the images. A brief review of the related literature, under the different approaches, is given below.

Generally, the main advantage of the AR model over the other models is that it is regenerative, that is, it represents all the information in an $N \times N$ image by two sets of parameters, one set containing a minimum number of parameters having most of the information while another set containing N^2 parameters, the so-called residuals, having the remaining information. Kashyap suggested that the residuals can be stored with minimum number of bits than the original image pixels without sacrificing any accuracy. With the use of stored parameters and the residual values, the original image can be reconstructed with good quality, whereas the textured images can be generated with the use of stored parameters of the model only and without any compromise in the quality of the image.

Several authors have shown a considerable attention on MRF and AR models, due to its simplicity, i.e. less computational complexity, finite memory or memory less and wide range of applications especially in texture analysis, segmentation, inpainting, reconstruction and in data mining, which searches and retrieve the images from the volume of database that contains images.

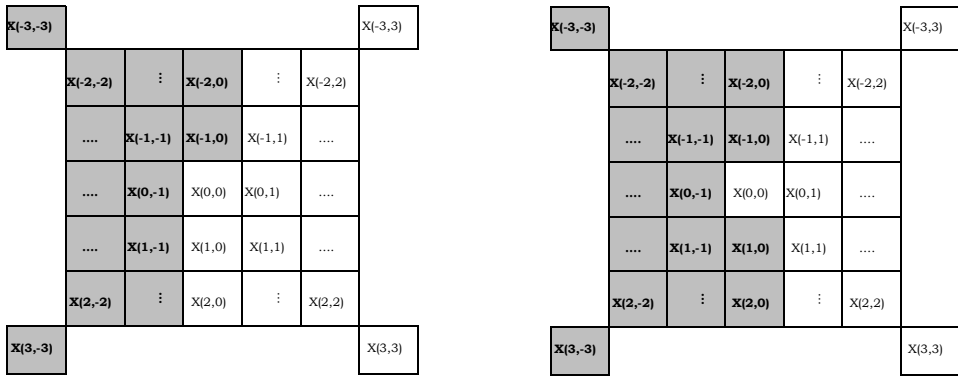
1.2 Problems in Selecting Models

There are different types of mathematical / statistical models. They can be broadly classified into Probabilistic models and Transformable model. The probabilistic model completely deals with the probability, random process, sampling, etc., whereas the transformable models deal with the quantization, i.e. the data in the spatial domain to frequency domain. Generally, the actual images are in the spatial domain. By transforming the images into frequency domain, the features can be extracted and it may be convenient for any other type of image processing.

The mathematical or statistical models can be classified into either linear or non-linear models. The linear model leads to a better results, if it is adopted for the homogeneous data. In terms of image, it may be texture image. The non-linear model is appropriate for the inhomogeneous data. In terms of image, it may be structure image. Hence, it is the important one that the models should be applied carefully; otherwise, it can lead to wrong results. If we employ a non-linear model on homogeneous data or texture image, surely it does not yield better results compared to that of linear models.

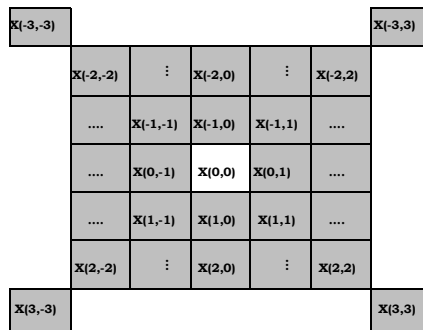
1.3 Types of Models

There are a number of probabilistic models. Broadly, they are categorized into three types of stochastic models such as Causal, Semi-causal and Non-causal.



(a) Causal

(b) Semicausal



(c) Noncausal

Figure 1.1 The canonical prediction regions.

The probabilistic models also can be classified into Time Series models and Stochastic models. The time series models deal with the forecasting and prediction, whereas the stochastic models deal with the Markov properties and the conditional probability.

The time series models are,

1. Autoregressive (AR)
2. Moving Average (MA)
3. Autoregressive Moving Average (ARMA)

4. Autoregressive Integrated Moving Average (ARIMA)

The stochastic models are,

1. Gaussian Random Field (GRF) models
2. Mixture Gaussian Random Field (MGRF) models
3. Markov Random Field (MRF) models
4. Gaussian Markov Random Field (GMRF) models
5. Hidden Markov Random Field (HMM) models
6. Gibbs Field (GF) models

Most of the above listed models deal with the homogeneous data or the texture type of images, while the Gibbs Field models are appropriate for analysing the heterogeneous data or structure type of images.

The transformation based models play a significant role in image processing. They yield better results for image compression, enhancement, etc. They are,

1. Fourier Transform (FT) model
2. Discrete Cosine Transform (DCT) model
3. Orthogonal Polynomial Transformation (OPT) model
4. Wavelet Transform model

It is the interesting topics in image processing that how to perform different types of image processing steps through a single statistical model. It is very useful to the researchers, scientist, designer of the image processing oriented softwares and post graduate students in Computer Science, especially those are working in the area of computer vision, signal processing, machine learning, data and knowledge discovery, and pattern recognition etc.

Throughout this book, it is discussed that the efficiency and effectiveness of applying the Gaussian Markov Random Field model on various types of images for different types of image processing, such as texture analysis – texture

characterization, identification, representation, classification; image compression; edge detection; restoration.

This book, also, discusses the appropriateness of the model and how to test the fitness of the model on the data or image; probability conditions and the randomness of the pixels in the image; various estimation techniques.

1.4. Summary

In this chapter, introduction of the various types of models, i.e. autoregressive, moving average, autoregressive moving average, autoregressive integrated moving average, and also the various stochastic models, such as Gaussian Random Field models, Mixture Gaussian Random Field models, Markov Random Field models, Gaussian Markov Random Field models, Hidden Markov Random Field models, Gibbs Field model have been discussed. Moreover, the transformation based models, namely Fourier Transform model, Discrete Cosine Transform model, Orthogonal Polynomial Transformation model, Wavelet Transform based model have been discussed. Also, the different types of models such as causal, semi-causal and non-causal models have been discussed, which are technically deals with the data regenerative process.

Randomness, Sampling and Quantization

2.1 Randomness, Sampling and Quantization

Generally, when capturing an image through a camera (digital or analog), there are many possibilities to register objects or images with the dust particles, that are moving randomly in any directions in the open space. Thus, the object or image registered in the camera is assumed to be a random process. Hence, the object or the image is assumed to be a random field. The object or image registered through a camera is a continuous function.

While scanning an image through a scanner, it scans the intensity of the light or brightness. Scanning means that it quantizing the intensity of the light or brightness of the object. Quantization means the digitization of the intensity in terms of decimal values (0 or 1 for binary image; 0 to 255 for gray-scale image; 0 to 16581375 ($255 \times 255 \times 255$)). The quantization process is a discrete. Thus, the intensity values in the digitized image are a discrete real valued function. The quantized intensity values are called as pixels. The scanned decimal values are stored in two-dimensional arrays. Hence, an image can be defined as follows:

An image can be defined as a two-dimensional array of real values in the spatial domain, whereas the image is defined as two-dimensional array of real or complex values in the frequency domain.

While a scanner scanning an image, it does not completely quantize the continuously registered intensity or brightness of the object or image. Instead, it scans in a uniform or periodic manner, which is discrete. This process tends to the sampling technique. Therefore, in general, the discrete models are considered for analysing or processing the images. It may be irrespective of either stochastic or transform based models.

The pixel values are assumed to be independent and identically distributed to a Gaussian random process or mixed process or even it may be a distribution free process. Thus, the images are assumed to be affected by a Gaussian random process.

Let X be a random variable that represents an intensity value with additive noise of a pixel at location (k, l) in a color image. The pixel $X(k, l) \in \mathbb{R}^3$ is a linear combination of three colors, such as red, green, and blue, i.e. $X(k, l) = [r(k, l), g(k, l), b(k, l)]^T$, where T represents the transformation of the vector. The mean intensity value of each color is represented by μ_r, μ_g and μ_b respectively, and the variance-covariance matrix is denoted by Σ . The multivariate normal density function of $X(k, l)$ is given by

$$\frac{1}{(\sqrt{2\pi})^3 |\Sigma|} \exp\left(-\frac{1}{2}(x - \mu)^T \Sigma^{-1} (x - \mu)\right) \quad (2.5)$$

The density function mentioned in Eq. (1) can be denoted as $n(x/\mu, \Sigma)$ and the distribution law as $N(\mu, \Sigma)$. The i -th diagonal element of the covariance matrix, σ_i is the variance of the i -th component of $X(k, l)$. The mean vector of each color of the pixels in the image is

$$\mu = E(X) = E \begin{bmatrix} X_r \\ X_g \\ X_b \end{bmatrix} = \begin{bmatrix} \mu_r \\ \mu_g \\ \mu_b \end{bmatrix}, \quad (2.6)$$

and the variance-covariance matrix is

$$\Sigma = \begin{bmatrix} \sigma_{rr} & \sigma_{rg}\rho & \sigma_{rb}\rho \\ \sigma_{gr}\rho & \sigma_{gg} & \sigma_{gb}\rho \\ \sigma_{br}\rho & \sigma_{bg}\rho & \sigma_{bb} \end{bmatrix} = \begin{bmatrix} \sigma_r^2 & \sigma_{rg}\rho & \sigma_{rb}\rho \\ \sigma_{gr}\rho & \sigma_g^2 & \sigma_{gb}\rho \\ \sigma_{br}\rho & \sigma_{bg}\rho & \sigma_b^2 \end{bmatrix} \quad (2.7)$$

where, σ_r^2 , σ_g^2 , and σ_b^2 are the variations among the intensity values of red, green, and blue colors respectively; σ_{rg} represents interaction between the red and green colors; similarly, σ_{rb} and σ_{gb} represent interaction between the corresponding colors; ρ represents correlation or interrelation between the corresponding color pixels. The covariance matrix Σ is symmetric and positive definite.

2.5 Summary

While capturing an image through a camera, the dust particles in the air space move randomly in any direction. Thus, the images are assumed to be *random field*. A scanner does not completely scans the intensity or brightness registered with the images; instead it scans the intensity or brightness of the images with time dependent and discrete state space manner. Thus, the *sampling concept is indirectly adapted* in the images. The scanned images are also known as a *discrete function*. While scanning an image, the brightness or intensity is quantized with decimal values. This process is known as quantization.

Image Model and Parameter Estimation

3.1 Introduction

3.3 Parameter Estimation of FRGMRF Model

3.4 Summary

This chapter discussed about a model with statistical properties, which describes the probability structure of a time series and in general any sequence of observations, is called stochastic process. The image to be analysed may be thought as one particular realization, produced by the underlying probability mechanism of the image under study. That is, in analysing an image we regard it as a realization of a stochastic process.

Prior information means that a prior knowledge about the data. A hyper-parameter is that the parameter of the distribution, which contains some prior information about the actual distribution. The hyper-parameter concept is discussed in the context of Bayesian theory.

The advantage of the present approach is that a number of parameters is fixed to four only and the order does not increase the computational complexity, since the estimation of these parameters is the same irrespective of the order of the model, and hence it increases efficiency of the model.

Chapter 4

Texture Analyses

2.2 Introduction

Texture analyses plays a noteworthy role in computer vision applications and image understanding, since most of the images contain textures. It includes the problems such as texture characterization, identification, description, representation, classification, texture segmentation, texture synthesis, object recognition and textured image compression. As the texture is a key feature for these problems, it must be properly represented, before analysing it. The proper representation of the structures of the textures will lead to the right path of texture analysis. The texture of a surface is characterized by properties such as fine, coarse, smooth, granulated, rippled, mottled, irregular, random, lineated, etc. Despite its ubiquity in scene analysis, there exists no precise definition for texture. Several authors have given the definition for textures in different ways based on its characteristics. Some of them are given below:

Picket has defined the texture as consisting of a large number of elements, each with some degree visibility, and on the whole, densely and evenly (possibly randomly) arranged over the field of views such that there is a distinct characteristic spatial repetitiveness in the pattern.

Vilnrotter et al. have described the texture as the pattern of the spatial arrangement of different intensities.

Ganesan and Bhattacharyya viewed textures as a composed of a large number of more or less ordered, similar elements or patterns.

Seetharaman proposed a definition for texture, which may be most appropriate for all types of textures.

Definition: A texture can be defined as a structure encompassing a large number of similar primitives or patterns that are scattered randomly or almost ordered with distinct characteristics of spatial repetitiveness.

4.8 Summary

In this Chapter, a new statistical approach based on the stochastic model and the autocorrelation function. The autocorrelation coefficient is derived from the model coefficients. The FRGMRF model based scheme proposes two texture descriptor: (i) *texnum*, the local descriptor and *textspectrum*, the global descriptor. Seetharaman introduced a decimal number to represent the textures that range from 0 to 200. These numbers uniquely represent the texture primitives. Totally, it has 201 components. Based on this *texnum*, the FRGMRF based scheme tests a good number of images formed with different types of textures. Also, two different sets of textured images are analysed in our scheme. The first one consists of four different types of textured images and the second one contains five different types of textured images. The FRGMRF model based scheme results the average classification is upto 94.30% and 93.57% for supervised and unsupervised classifications respectively for test image merged with four different types of textured images. The supervised and unsupervised algorithms are employed to classify the test image, which is formed with five different types of textured images, the average correct classification is up to 86.2% and 85.80%.

Chapter 5

Edge Detection

5.2 Image Smoothing

This section focuses on the estimation of the input image surface. A gray-level input image $f_i(x, y)$ with size $L \times L$ is considered and the pixel values are in the range from 0 to 255. The image is partitioned into various equal size blocks, each of size $M \times M$ ($M < L$) with the pixel of interest at the centre. The parameters K , α , θ and ϕ of the FRGMRF model are estimated, as discussed in Chapter 3. With the use of estimated parameters, the model coefficients Γ_r s in equation (3.1) are calculated and are used to estimate the centre pixel value in the block. The FRGMRF model filters the features of the pixel of interest located at the center of the window with size $M \times M$. The estimated value is stored in another image array. This procedure is continued to cover the entire image by sliding the block in the raster scan fashion, and the estimated image surface of the input image is obtained. Now, the estimated surface is called as smoothed image $f_s(x, y)$, which is shown in Figure 5.1, with their corresponding original images.

5.3 Edge Magnitude and Direction

The edge magnitude is defined as the difference between the original image $f_i(x, y)$ and smoothed image $f_s(x, y)$. At each pixel location (x, y) , the edge magnitude is measured by taking the absolute value of the difference between the pixels in the corresponding locations of original and smoothed images.

That is,

$$f_d(x, y) = f_i(x, y) - f_s(x, y)$$

$$M(x, y) = |f_d(x, y)| \quad (5.1)$$

According to statistical theory, the difference between the actual and estimated values is known as *residual*. In the scheme suggested by Seetharaman, the residual represents the features, such as edges, textures, creases and boundaries of the untextured images, which are not captured by the model when measuring the smooth image surface of the input image. As indicated Chen, the highest gradient between the original and smoothed image surfaces represents the edges. The noises have zero-crossings while those image surfaces are tangent, but with very minimum gradient. If the smoothed image is parallel to the original with slight perplexity, then it indicates the presence of textures. These geometrical representations of the features in the given image are given in Figure 5.2.

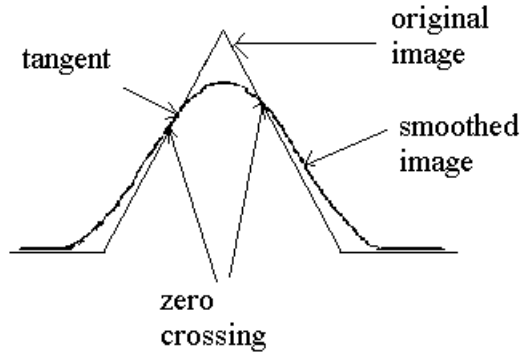


Figure 5.2. Geometrical representation of the features in the image.

For the residual image, the global statistic, mean \bar{x} and standard deviation σ are calculated and applied in equation (5.2) to measure the confidence limit at the required significance level.

$$Upper\ Confidence\ Limit = \bar{X} + t \frac{\sigma}{\sqrt{n}} \quad (5.2)$$

where, t is level of significance and n is the number of pixels in the image. Then each value in the residual image is compared

with the confidence limit. If the pixel value is greater than or equal to the confidence limit, then it is squared; otherwise, the pixel value is replaced with zero.

That is,

$$f_e(x, y) = \begin{cases} [f_d(x, y)]^2 & ; \text{if } f_d(x, y) \geq \text{UCL} \\ 0 & ; \text{Otherwise} \end{cases}$$

To illustrate this concept, consider the sub-images of size 3×3 as shown in Figure 5.3(a) and 5.4(a) that are taken as the original form of background scenes or texture and untextured regions. These image regions are subjected to the proposed smoothing process and the results of smoothing are presented in Figure 5.3(b) and 5.4(b) respectively. As discussed above, we estimate the pixel values. The original and estimated pixel values are plotted geometrically to describe the structure of the various features in the image. The plotted geometric structure is shown in Figure 5.5. The following information can be observed from Figure 5.5(a).

- (i) The highest gradient (corresponding to 3rd x-ordinate value) represents the fine edges, while the moderate gradient (corresponding to 2nd and 5th x-ordinate values) represents the weak edges.
- (ii) The tangent with a very small gradient between the original and smoothed images indicates the noises.
- (iii) The slight wavering parallel lines with a minimum gradient (corresponding to 7th, 8th and 9th x-ordinate values) represent the textures or background scenes.

Figure 5.5(b) exposes the pixels that falls outside the Upper Confidence Limit (UCL). Its enhanced value represents the region of thick edge, and is shown in Figure 5.6.

(a) (b) (c)

Figure 5.3.Edge magnitude for background images; (a) original image;(b) smoothed image; (c) residual image.



Figure 5.6.Edge region extracted image.



(a)

(b)

Figure 5.7.Edge map using the proposed method.



(a)

(b)

Figure 5.8.Edge map: Figure 5.8(a) and 5.8(b) are superimposed on its original images.

5.6 Comparison of Various Edge Maps

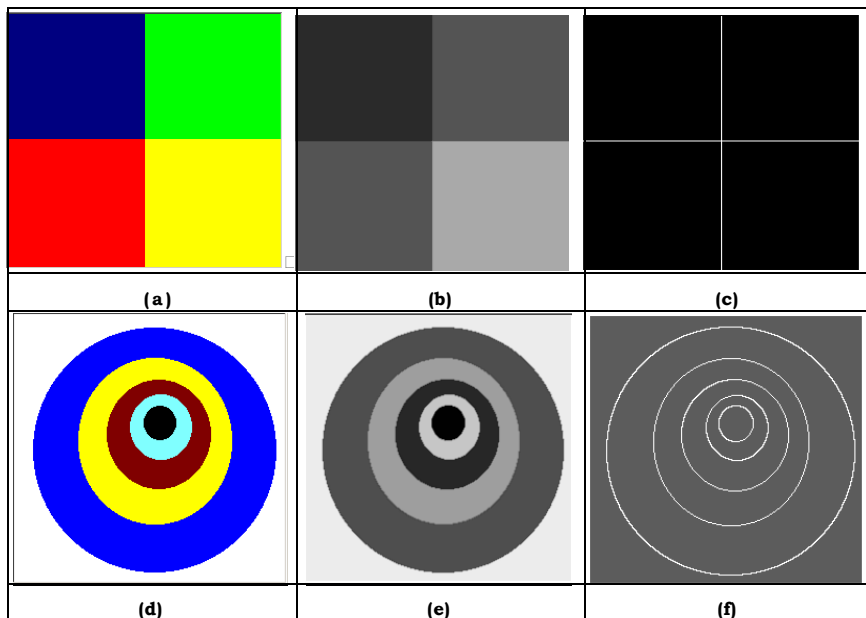


Figure 5.10. (a) and (d) are original colour images of concentric circle and square with size 256×256 ; (b) and (e) are smoothed grayscale images; (c) and (f) are obtained edge maps.

Image Restoration

7.2 Damaged and Prior-Information Blocks Identification

To identify the damaged blocks in the corrupted input image, the image is divided into various blocks of size 8×8 , and the statistics mean and variance are computed on each block and that are applied in the confidence interval expressed in equation (7.1). The value of σ^2 is compared to that of the lower and upper limits as in equation (7.1). If the σ^2 is less than the lower point, then the block is assumed to be damaged because most values in the damaged block are nearer close or same, so there exist less variation among the pixels in the damaged block; if the σ^2 is greater than the upper point, then the block is assumed to be partially damaged (combination of damaged and undamaged blocks) because the pixel values are in the low and high ranges (i.e. 0 to 255), so there exists high variation; otherwise, the block is treated as undamaged because the pixels belong to the group of moderate intensity values, so there exist moderate variation. In the case of the partially damaged block, the region growing technique is adopted to identify the boundary of the damaged block.

$$\frac{ns^2}{\chi_n^2(\alpha/2)} \leq \sigma^2 \leq \frac{ns^2}{\chi_n^2(1-\alpha/2)} \quad (7.1)$$

$$\text{where, } s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2$$

$$\mu = \frac{1}{n} \sum_{i=1}^n x_i$$

$\chi_n^2(\alpha/2)$ -Chi-square distribution with n degrees of freedom and significance level α ; $\chi_n^2(\alpha/2)$ is the lower α point and $\chi_n^2(1-\alpha/2)$ is the upper α point.

The searching and matching methodology of the prior information is illustrated in Figures 7.1 and 7.2. A detailed procedure to identify the damaged block and the prior information is presented in an algorithmic form in section 7.5.

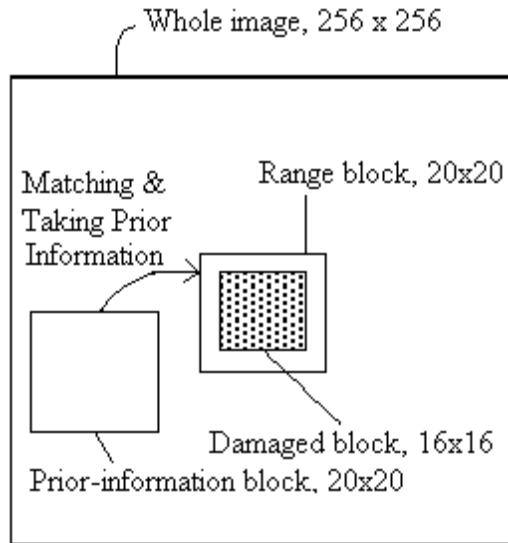


Figure 7.1. Structure of the searching technique

7.4 Image Restoration

The methodology adopted to restore the pixels of the damaged block is illustrated in Figure 7.3.

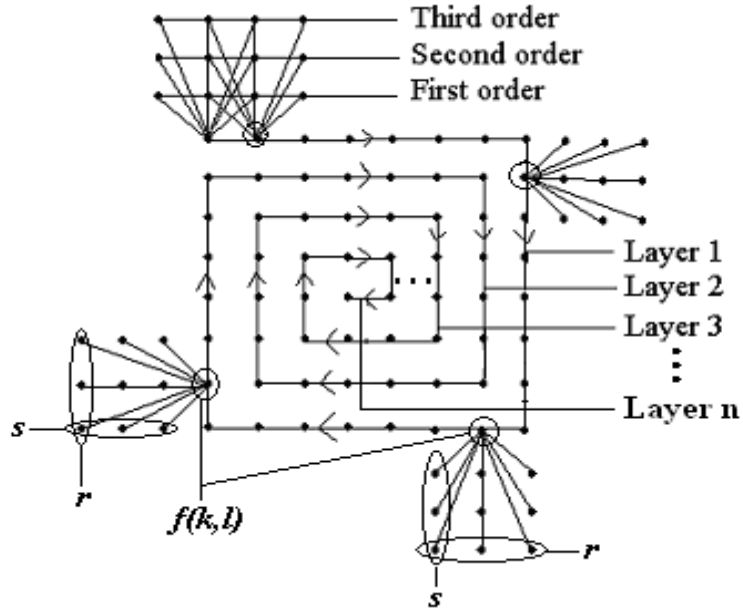


Figure 7.3. Structure and restoration technique of damaged block

In order to implement the concept depicted in Figure 7.3, the model in equation (7.2) is restructured as in equation (7.2) if $r < k$; $r > k$,

$$f(k,l) = \sum_r \left(\frac{1}{3} \sum_s \Gamma_{|r|} g(k+r, l+s) \right) + \varepsilon(k,l) \quad (7.2)$$

with constraints,

$$\begin{cases} -1 \leq s \leq 1 \\ -1 \leq r \leq -3 \end{cases} \text{ if } r < k \quad \text{and} \quad \begin{cases} -1 \leq s \leq 1 \\ 1 \leq r \leq 3 \end{cases} \text{ if } r > k$$

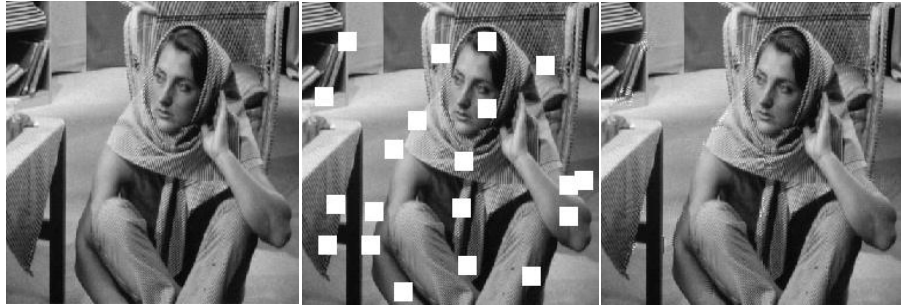
and the same model is restructured as in equation (7.3) if $s < l$; $s > l$,

$$f(k,l) = \sum_s \left(\frac{1}{3} \sum_r \Gamma_{|s|} g(k+r, l+s) \right) + \varepsilon(k,l) \quad (7.3)$$

with constraints,

$$\begin{cases} -1 \leq r \leq 1 \\ -1 \leq s \leq -3 \quad \text{if } s < l \end{cases} \quad \text{and} \quad \begin{cases} -1 \leq r \leq 1 \\ 1 \leq s \leq 3 \quad \text{if } s > l \end{cases}$$

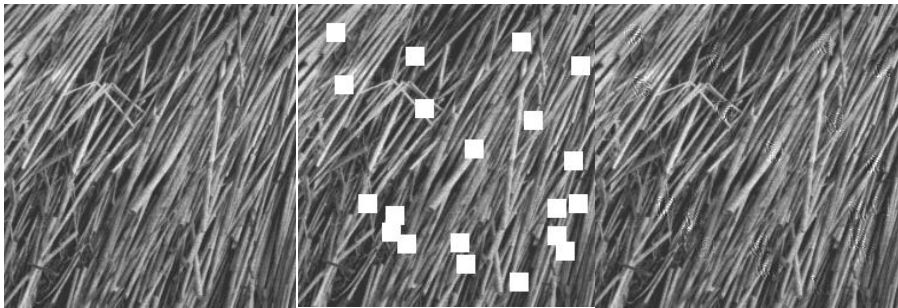
where, $\Gamma_{(s)} = \frac{K \sin(r\theta) \cos(r\phi)}{\alpha^r}$. (7.4)



(a)

(e)

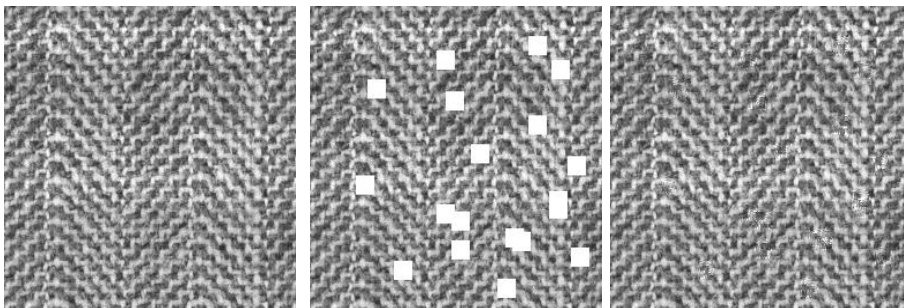
(i)



(b)

(f)

(j)



(c)

(g)

(k)

Figure 7.6. Column 1: original images; Column 2: damaged images; Column 3: restored images

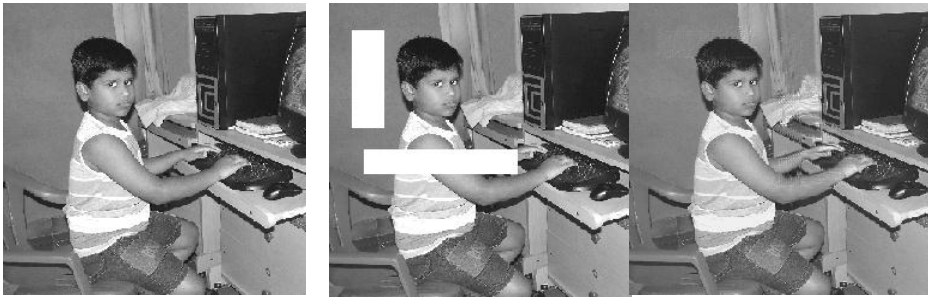


(a)

(b)

(c)

Figure 7.7. Lena Image: (a)-Original image; (b)-damaged image; (c)-restored image.



(a)

(b)

(c)

Figure 7.8. Darani Image: (a)-Original image; (b)-damaged image; (c)-restored image.

Chapter 8

Conclusion

8.2 Advantages of FRGMRF Model Based Scheme

The proposed common framework has some features, which are described as follows. Generally, the model selection is a major problem in predictive coding technique, since it takes considerable amount of time to choose the appropriate model. The FRGMRF model has an infinite structure with a finite number of parameters, i.e. the number of parameters is fixed to four, not depending upon the order of the model. In the FRGMRF model, the order does not increase the computational complexity because the estimation of the parameters is the same irrespective of the order of the model. Thus, it increases the efficiency of the model. In the case of texture representation, the global descriptor, viz. the texspectrum is having only 201 components, which is much lesser than the existing schemes. Hence, the scheme introduced by Seetharaman is considerably reduce the computational time complexity. The already existing model based approach, such as white noise, autoregressive finite order and infinite order autoregressive models can be treated as special cases of the FRGMRF model. The proposed edge detection scheme detects all types of edges, and the edges in all directions, at the same time it does not detect the weak and spurious edges, because of it smoothesthe abrupt changes minimally across the edges.

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